Single Qubit Arbitrary Unitary Synthesis Using Photonic Spectral Encoding

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Motivation: The efficient experimental implementation of an arbitrary unitary transformation is central to future quantum technologies to aid the development of fields such as quantum computing, communications and cryptography. Optical techniques offer an especially promising framework owing to both, the relative ease with which light states can be transformed, as well as the ready availability of broadband compact and inexpensive optical components which are very well-suited to photonic integration over many modes.

Background: Several recent efforts have discussed methodologies which leverage the properties of light. For instance, in [1], a succession of reflections on deformable mirrors separated by Fourier transforms is shown to perform any unitary spatial transformation on an input mode of light. Several works by the groups of J. M. Merolla and J. Capmany discuss spectral encoding using electro-optic phase modulation to perform non-diagonal gates on a qubit as an enabling technique for applications such as quantum key distribution (QKD) ([2],[3]). More recently, the idea of "Spectral Linear Optical Quantum Computation" (spectral LOQC) has been proposed [4], where frequency-encoded photonic qubits are shown to be a highly-promising resource for scalable quantum information processing. In this scheme, the concept of dual-rail encoding has been used to encode a qubit over two spectral modes. Although the approach is well-suited for scaling to multiple qubits, at least 8 modes ("guard bands") are needed to isolate each qubit from the next, thereby leaving a large number of modes unavailable for encoding.

In this work, we develop a mathematical framework for single-qubit unitary synthesis in a 4-dimensional spectral mode space which allows for spectral encoding on light. Following the treatment in [4], any unitary can be synthesized using just two components - pulse-shapers ($\hat{PS}$) and electro-optic phase modulators ($\hat{EOPM}$) in an alternating sequence. The $\hat{PS}$ allows the phase of each mode to be individually manipulated, and thus, has the form Diag$\{e^{i\varphi_1},...,e^{i\varphi_M}\}$ where the M phases are individually-applied to the M frequency modes. On the other hand, the $\hat{EOPM}$ couples different frequency modes and can act in the high-bandwidth regime. It has the form $\hat{F}$Diag$\{e^{i\theta_1},...,e^{i\theta_M}\}\hat{F}^{\dagger}$, where $\hat{F}$ denotes the M x M discrete Fourier transform matrix.

Results: We first show that by choosing to encode in the even and odd modes of a 4-dimensional spectral mode space, the Hadamard gate can be implemented exactly using just 3 components (2 PS and 1 EOPM, in the sequence $(\hat{PS}_2)(\hat{EOPM})(\hat{PS}_1)$) in 4 modes, as opposed to the 4 components (2 PS and 2 EOPMs, in the sequence $(\hat{EOPM}_2)(\hat{PS}_2)(\hat{EOPM}_1)(\hat{PS}_1)$) proposed in [4], where a minimum of 8 modes are required for a good gate performance. Next, we show that 3 PS and 2 EOPMs are sufficient to synthesize an arbitrary single-qubit unitary in the following form:

$$\hat{V} = (\hat{PS}_3)(\hat{EOPM}_2)(\hat{PS}_2)(\hat{EOPM}_1)(\hat{PS}_1) \tag{1}$$

Finally, we address the question of whether an arbitrary single-qubit unitary can be realized using just the three components used to synthesize the Hadamard gate.

Oulook & Prospects: Since the ability to synthesize an arbitrary single-qubit unitary is sufficient to implement most applications in quantum communications and cryptography, the present formalism offers a complete framework for the same. At the same time, a judicious choice of encoding also enables realizing such unitaries with a minimum number of components, which could prove useful for future designs involving chip-scale integration. Future work will investigate the possibility of synthesizing arbitrary two-qubit unitaries, paving the way for universal quantum computing.

References

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