Investigating the optimality of ancilla-assisted linear optical Bell measurements

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Due to its experimental and theoretical simplicity, linear quantum optics has proved to be a promising route for the early implementation of important quantum communication protocols [1]—including quantum teleportation, dense coding and entanglement swapping. An essential step in these protocols is the *Bell measurement* (BM), a projective measurement onto a basis of two-qubit maximally entangled states, the Bell states. Here we are concerned with *unambiguous* BM, i.e. a measurement that correctly identifies some the Bell states without errors, but gives an inconclusive answer with a non-zero probability. Lütkenhaus *et al.* [2] showed long ago the impossibility of perfect linear optical Bell measurement for dual-rail photonic qubits. In a following result, Calsamglia and Lütkenhaus bound the success probability P_{succ} of the no-ancilla case to 50% [3]. However in the last decade Grice [4] and Ewert and van Loock [5] found linear optical networks achieving near-unit efficiency unambiguous Bell state discrimination, when fed with increasingly complex ancillary states. However, except for the vacuum ancilla case [3], the optimality of these schemes is unknown.

Here, the optimality of these networks is investigated through analytical and numerical means. We show an analytical upper bound to the success probability for interferometers that preserve the polarization of the input photons. While this restriction is not motivated by experimental realities, instead being a consequence of the proof technique, we show that the bound is saturated by both Grice's and Evert-van Loock's strategies. Furthermore, we obtain a link between the complexity of the ancilla states and the scaling of the performances of the discrimination. Among other states, we compute this bound for networks with k ancillary single photons, or equivalently $\frac{k}{2}$ ancillary Bell pairs. We bound the success probability of these cases to $P_{\text{succ}} \leq 1 - O(\frac{1}{\sqrt{k}})$, giving interesting insights into the reason why the big GHZ-like states that appear in the schemes of [4] are needed to attain their linear $P_{\text{succ}} \leq 1 - O\left(\frac{1}{k}\right)$ scaling.

In the second part of the work, we show a computer-aided approach to the optimization of such measurement schemes for generic interferometers (not constrained to be polarization-preserving), by simulating an optical network supplied with various kinds of ancillary input states. For each ancilla we want to analyze, and for each input Bell state, we generate a symbolic expression for the probability amplitudes of all output events in terms of the unitary matrix U representing the network. Those functions, along with their gradient with respect to the U entries, are symbolically optimized exploiting physical and mathematical symmetries in order to reduce the number of operation needed. Then, a constrained numerical optimization using a nonlinear method is performed. With this program we numerically confirm the optimality of known small schemes. We also numerically investigate the usefulness of single photons, additional Bell pairs and GHZ and W states, obtaining $P_{\text{succ}} = \frac{3}{4}$ as the highest success probability we achieve with small (≤ 10 modes, ≤ 6 photons) networks.

Associated paper pre-print: arXiv:1806.01243 . *Source code available at:* arxiv.org/src/1806.01243/anc

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